Simultaneous Equations

Before You Watch

This topic follows directly on from Linear Equations. It covers how to take two linear equations and solve them together to get a single value for the two variables (which are usually x and y, but could be any two letters). Thinking about this process graphically, we are finding the point where the two lines intercept. This is called solving simultaneous equations. Keep in mind that in Australia this topic is known as simultaneous equations, but in other countries it is often referred to as “solving a system of equations”.

If you haven't watched **Linear Equations** yet, take a look at that video first, then come back.

The Video Content

Linear equations are used in a variety of applications. But what if you’re asked to solve two linear equations simultaneously? We’ll show you what this means, and how to do it. Let’s start with an example.

Say we’ve been asked to solve these equations simultaneously:

$$y = 3x + 2$$
$$2y + x = 26$$

So what does ‘solving simultaneously’ mean?

Step 1 Understand the question

A single linear equation has an infinite number of solutions, and these solutions can be represented by a line when we graph them. But if there are two linear equations, we can solve them together. This will give us a single solution, where the two lines cross!

That's what solving simultaneously means: to take two equations – each of which has an infinite number of solutions – and find a point which is a solution to both of them.

Step 2 Develop a plan

How do we do this? Let's go back to the two equations. Each of them has two letters. To solve this, we need to combine these two equations, to get an equation with only one letter in it. If an equation has only one letter, we can solve it to find a value for that letter.

Step 3 Carry out the plan

It helps to number the equations, so they are easier to keep track of:

$y = 3x + 6$ **(1)**

$2y + x = 26$ **(2)**

Equation **(1)** tells us that y is equal to $3x + 6$. So, taking this value for y from equation **(1)** and substituting it into equation **(2)**:

$2\ (3x + 6) + x = 26$

Now there are only x's! Let's expand the brackets and simplify:

$6x + 12 + x = 26$	now bring the x's together
$7x + 12 = 26$	now take away 12 from both sides
$7x = 14$	now divide both sides by 7
$x = 2$	now we know that $x = 2$

Next, to find the value of y, substitute $x = 2$ into one of the original equations, say equation **(1)**, which is $y = 3x + 6$:

$y = 3\ (2) + 6$

$y = 6 + 6$

$y = 12$

Now we know that $y = 12$. And that's our solution:

$x = 2$ and $y = 12$.

Step 4 Reality Check

To check that the answer is correct, substitute x = 2 into both the original equations to see if y = 12.

Yes, this works in both of them, so we know we have the correct solution.

Some Practice Questions

Solve simultaneously:

1. $7 = 3x + 2y$, $4x - y = 3$
2. $11 = 7n - 4r$, $5 + 12n + r = 11$
3. $0.5p = r + 7$, $2p = 3r + 7$
4. $3 = x / y$, $y = x + 4$

Answers

1. x = 13 / 11, y = 19 / 11
2. n = 81 / 52, r = -207 / 104
3. p = -28, r = -21
4. x = -6, y = -2

Take a look at the working out for each answer **here**.

Now What?

Here we've shown the simplest way of solving two linear equations simultaneously. In general, any two equations with the same two unknowns can be solved simultaneously. Bear in mind, however, that depending on the equations, that's sometimes not possible. Let's think about it graphically: each equation is a line, and if the two lines cross, you can solve them. If they don't cross, you won't be able to find a real solution.

Moving beyond two equations, what if there are three, or four equations? If you have more equations, you can find more unknowns.

Once you are comfortable with solving two equations simultaneously, consider giving three equations a go at:

- https://www.khanacademy.org/math/algebra2/systems_eq_ineq/fancier_systems_precalc/v/systems-of-three-variables

You should also consider looking at how to solve other equations, such as **quadratic equations** or equations with **exponentials**.

But When Am I Going To Use This?

Linear equations are very commonly used in everyday life to model situations. So when two things are both modelled by a linear equation, we can solve them together (simultaneously) to find out important information.

A common example of this is in finance with the analysis of revenue and costs. The revenue for a company from a product (say R) is simply the price of the product (let's say \$30) *multiplied by* how many they sell (n), so:

$$R = 30n$$

This relationship can be represented by a straight line. Similarly, the cost of that product (C) is a fixed starting amount (the development cost, staff wages and so on) *plus* whatever the cost of making the product is *multiplied by* how many are sold.

Let's say there is a fixed monthly cost of $1500 and the product costs $10 to make. That means the costs per month are:

$$C = 1500 + 10n$$

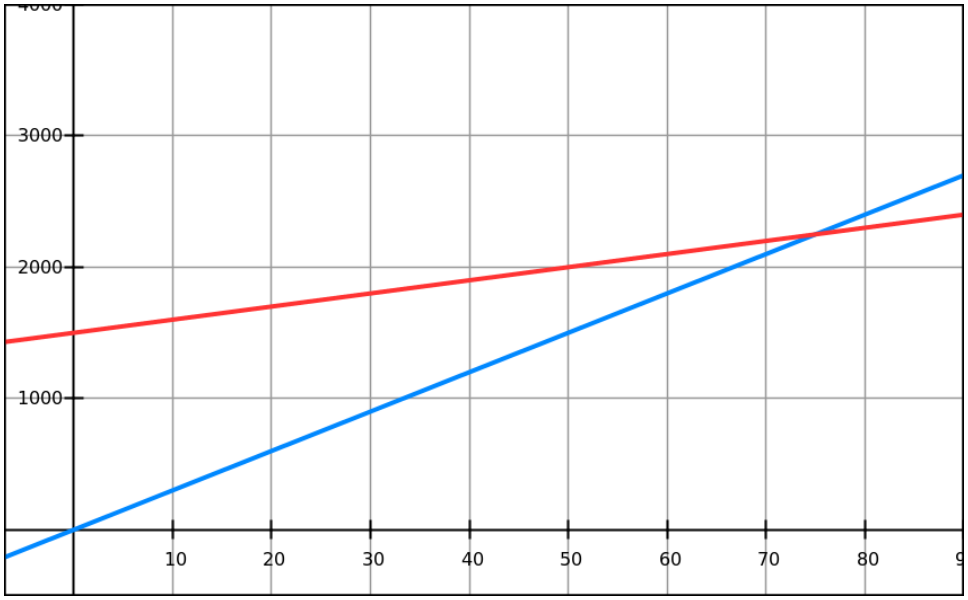
This can also be represented as a straight line. So when revenue is equal to costs ($R = C$), the company is breaking even.

By solving these two equations simultaneously:

$$C = 30n$$

$$C = 1500 + 10n$$

we can determine how many products, n, must be sold per month for the company to break even. See if you can work it out, the answer is $n = 75$. A graph of the functions is given below.



This is just a single example. Some other examples can be found at:

- http://www.shelovesmath.com/algebra/intermediate-algebra/systems-of-linear-equations/

Other Links

She Loves Maths has a wide range of pages covering many mathematical topics across all levels. The pages cover each topic using practical, relatable examples. This content deals with systems of linear equations.

- http://www.shelovesmath.com/algebra/intermediate-algebra/systems-of-linear-equations/

Maths is Fun provides a clear summary of solving equations and explains the different terminologies used. It also offers several questions to practise on.

- http://www.mathsisfun.com/algebra/systems-linear-equations.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link is to a chapter dedicated to solving equations, with over a dozen videos and several quizzes.

- https://www.khanacademy.org/math/algebra2/systems_eq_ineq/systems_tutorial_precalc/v/trolls-tolls-and-systems-of-equations

Patrick JMT (Just Maths Tutorials) has many video tutorials covering a large range of mathematical concepts. The content below demonstrates three different techniques for solving a system of linear equations. Any of these methods can be used to solve any system of equations but, depending on the exact question, one strategy may be easier to use than the others.

- http://patrickjmt.com/row-reducing-a-linear-system-of-equations/
- http://patrickjmt.com/solving-a-linear-system-of-equations-using-elimination-by-addition/
- http://patrickjmt.com/solving-a-linear-system-of-equations-using-substitution/