

### Before You Watch

This topic covers how to solve a particular type of equation called a quadratic equation.

To get the most out of this content, you'll need to be comfortable with the rearrangement of algebraic equations, as well as indices as they are applied to algebra. To brush up on your skills in these areas, take a look at the [Introduction to Algebra](#) and [Indices Laws](#) videos first, then come back.

### The Video Content

You will often see equations that look like this:

- $2x^2 - 1 = -x$
- $k^2 + 6k + 8 = 10$
- $p^2 - 2p = 0$

These equations are common in the real world. They help us figure out things like the side length of a square when we are given its area, or the trajectory of a ball when it is thrown.

These equations are called quadratic equations and they only have one unknown. In the above examples the unknowns are  $x$ ,  $k$  and  $p$ . When they are expanded, they have a squared term in their unknowns but don't have any powers higher than 2.

Let's look at the first example.

Solve the equation:

$$2x^2 - 1 = -x$$

So, what is the question asking?

### **Step 1 Understand the question**

When we are asked to solve a quadratic equation, we simply have to find the value of  $x$  which makes the equation true. Several methods can be used to do this, however, the most reliable is the quadratic formula. Therefore, using the formula is our plan!

#### ***Did you know?***

Another method of solving quadratics often taught in schools is to move all the terms onto one side of the equation, so only a 0 is left on the other side, then factorise the equation. While this method works sometimes, and helps with understanding quadratic equations, it is unreliable. That is, it is often very difficult to see the correct way to factorise a quadratic equation, unless the question has been crafted to make the factorisation easy. For this reason, in practise most mathematicians will simply go straight to the quadratic formula, the strategy we are using here.

### **Step 2 Develop a plan**

What is the quadratic formula?

The quadratic formula is a rule that allows us to solve any quadratic equation.

This equation is in its general form:

$$ax^2 + bx + c = 0$$

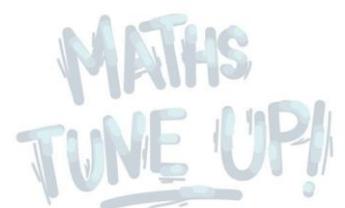
The quadratic formula looks like this:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now we can carry out the plan by using the formula.

### **Step 3 Carry out the plan**

First, to make sure the equation is in the general form, put everything on one side of the equals sign, leaving just a zero on the other side.



In our example:

$$2x^2 - 1 = -x$$

the easiest way to do that is to add  $x$  to both sides:

$$2x^2 - 1 + x = -x + x$$

giving us:

$$2x^2 - 1 + x = 0$$

Next, order the terms in decreasing order of the power of the letter. That is, put the  $x$  squared term first, then the  $x$  term, then the constant. Then compare this equation to the general form of the quadratic equation. To do this, write one above the other:

$$2x^2 + x - 1 = 0$$

$$ax^2 + bx + c = 0$$

By comparing these two equations we can see that:

$a$  is equal to 2

$b$  is equal to 1

$c$  is equal to -1

Now we substitute these values into the quadratic formula, which is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since we know that:

$$a = 2$$

$$b = 1$$

$$c = -1$$

we can write:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

which equals:

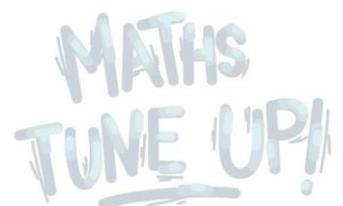
$$x = \frac{-1 \pm \sqrt{1 - (-8)}}{4}$$

The double negative becomes a positive, and so adding these terms gives:

$$x = \frac{-1 \pm \sqrt{9}}{4}$$

The only step left is to produce the solutions. Since the square root of 9 is 3, the quadratic formula gives us:

$$x = \frac{-1 \pm 3}{4}$$



Because of the  $\pm$  sign, we have two possible solutions – one when we add 3 and another when we subtract it:

$$\begin{array}{ll} x = (-1 + 3) / 4 & \text{or} \quad x = (-1 - 3) / 4 \\ x = 2 / 4 & \text{or} \quad x = -4 / 4 \\ x = \frac{1}{2} & \text{or} \quad x = -1 \end{array}$$

That's how we arrive at the two solutions:  $x = 1 / 2$  or  $x = -1$ .

#### **Step 4 Reality check**

To check that these answers are reasonable, substitute these values into the original equation:

$$2x^2 - 1 = -x$$

Does the left side of the equation equal the right side? Try it!

First, substitute  $x = \frac{1}{2}$  into the above equation:

$$2\left(\frac{1}{2}\right)^2 - 1 = -\frac{1}{2}$$

Then simplify it:

$$\begin{array}{l} 2\left(\frac{1}{4}\right) - 1 = -\frac{1}{2} \\ \frac{1}{2} - 1 = -\frac{1}{2} \\ -\frac{1}{2} = -\frac{1}{2} \end{array}$$

The last line is obviously true; the left side equals the right side, therefore we know that  $\frac{1}{2}$  is a correct solution.

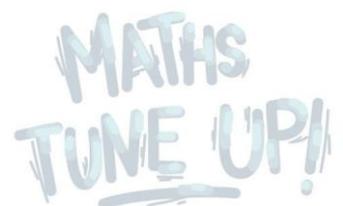
Second, substitute  $x = -1$  into the equation:

$$2(-1)^2 - 1 = -(-1)$$

Then simplify it:

$$\begin{array}{l} 2(1) - 1 = 1 \\ 2 - 1 = 1 \\ 1 = 1 \end{array}$$

Once again, the left and right sides of the equations match up, which confirms that the other solution is  $x = -1$ .



Quadratic equations in general have two solutions, but they cannot have more than two solutions. It is, however, possible to have only one solution, when the part inside the square root symbol of the quadratic formula is equal to zero.

## Some Practice Questions

1.  $k^2 + 6k + 8 = 10$

2.  $p^2 - 2p = 0$

3.  $f^2 - 3f - 10 = 0$

4.  $2y^2 + 4 = 9y$

Answers

1.  $k = -6.3$  or  $k = 0.3$

2.  $p = 0$  or  $p = 2$

3.  $f = 5$  or  $f = -2$

4.  $y = 4$  or  $y = 0.5$

Take a look at the working out for each answer [here](#).

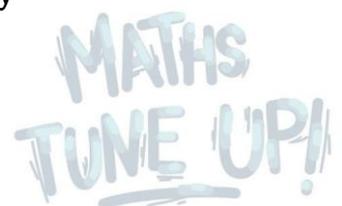
## Now What?

You may find it helpful to follow through and see how the quadratic formula is created.

You can see the derivation in the **Maths is Fun** website at

<http://www.mathsisfun.com/algebra/quadratic-equation-derivation.html>. It is not necessary to know this derivation, but it might assist your understanding. Another option for exploring the derivation of the quadratic formula is the video by

**PatrickJMT** at <http://patrickjmt.com/deriving-the-quadratic-formula/>.



Now that you're familiar with the use of the quadratic equation, why not check your skills in some of the areas covered by the other algebra videos? **Factorisation of Algebraic Expressions** is particularly relevant to the alternative method of solving quadratic equations. Others such as **Algebraic Fractions** and **Simultaneous Equations** will also help develop your abilities in algebra.

## *But When Am I Going To Use This?*

Quadratic equations are very important as they are useful in understanding situations such as movement under constant acceleration (like gravity), a thrown ball if we ignore wind resistance, or pendulums or weights on springs. In fact, the quadratic equation is the base level solution for all stable systems, so it's one of the most widely studied systems in physics and engineering. Some stable systems that are often approximated using a quadratic equation include an atom's position in a molecule or solid, a thermostat attempting to maintain a constant temperature, or even a child on a swing.

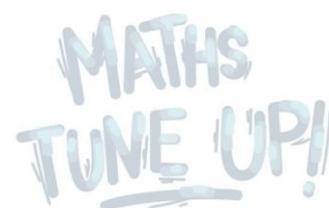
## *Other Links*

As well as showing the derivation of the quadratic formula, **Maths is Fun** has a great applet where you can enter the formula for any quadratic equation. It then graphs the corresponding parabola for that quadratic and solves it for you. This is useful because it helps you visualise the solutions to the quadratic as positions on a parabola. However, keep in mind that you will still need to be able to solve the quadratic equation yourself.

- <http://www.mathsisfun.com/quadratic-equation-solver.html>

It also has worked solutions and sample questions.

- <http://www.mathsisfun.com/algebra/quadratic-equation.html>



**Mathportal** has another great applet to help develop your understanding of the quadratic equation and how to solve it. It allows you to input any quadratic equation, and shows a worked step-by-step solution using either the quadratic formula, or another method known as completing the square.

- <http://www.mathportal.org/calculators/solving-equations/quadratic-equation-solver.php>

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. It has a chapter dedicated to solving quadratic equations using the formula. This includes eight videos and a summarising test at the end.

- <https://www.khanacademy.org/math/algebra/quadratics/quadratic-formula/v/using-the-quadratic-formula>

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Here are two videos relevant to this topic: the first covers the use of the quadratic formula, and the second is the first in a series of videos showing example questions.

- <http://patrickjmt.com/using-the-quadratic-formula/>
- <http://patrickjmt.com/solving-quadratic-equations-using-the-quadratic-formula-ex-1/>

