Negative and Fractional Indices

Before You Watch

This topic builds directly on the work done in Indices Laws, and also introduces new laws. Make sure you are familiar with the following rules of indices before starting this topic:

- $a^n \times a^m = a^{(n+m)}$
- $a^n / a^m = a^{(n-m)}$
- $(a^n)^m = a^{(n \times m)}$

Confident in the use of these laws of indices? Then you're ready! If not, watch **Indices Laws** first, then come back.

The Video Content

This content explains the mathematical meaning of indices, or powers, that are either zero, negative, or a fraction. It builds on the previous topic, so if anything doesn't make sense, refer to that video, then return.

To understand the meaning of something to the power of zero, let's consider:

$$t^2 / t^2$$

Using our index laws, we know this is:

$$t^2 / t^2 = t^{(2-2)}$$

which is:

$$t^{(2-2)} = t^0$$

We also know it is equal to 1, because anything divided by itself is equal to 1:

$$t^2 / t^2 = 1$$

Therefore:

$$t^0 = 1$$

So, anything to the power of 0 is 1.

Let's look at **negative indices**. A negative power means one over the positive power. For example:

$$p^{-6} = 1 / p^6$$

Consider:

$$f^3 \times f^{-2}$$

This can be solved in two ways. We know when we multiply letters with powers, we add the powers:

$$f^3 \times f^{-2} = f^{(3+(-2))}$$

Therefore:

$$f^3 \times f^{-2} = f^{(3-2)} = f$$

Another way is to use our new meaning of negative powers. We know that:

$$f^{-2} = 1 / f^2$$

Therefore:

$$f^3 \times f^{-2} = f^3 \times 1 / f^2$$

which is:

$$f^3 \times 1 / f^2 = f^3 / f^2$$

When dividing letters with powers, we subtract the powers, so this is:

$$f^3 / f^2 = f^{(3-2)}$$

which is:

$$f^{(3-2)} = f$$

Either way we get the same answer.

Now let's look at **fractional indices**.

A fractional index of $1/n$ is the same as taking the nth root. For example:

$$y^{1/3} = \sqrt[3]{y}$$

Did you know?

This section on fractional indices applies to fractional indices of positive numbers. It is not so straight-forward for negative numbers. Try taking your calculator and finding:

$$\sqrt{-1} = (-1)^{½}$$

You should get an error. This is a branch of mathematics called complex numbers, or imaginary numbers.

You may come across complex numbers later on, but for now just keep in mind that this section works for positive numbers only.

Another example:

$$(d^2)^{1/2}$$

Here, we could use the index laws for powers of powers. This means:

$$(d^2)^{1/2} = d^{2 \times 1/2}$$

which is:

$$d^{2 \times ½} = d$$

Or we could use the law for fractional indices, so:

$$(d^2)^{1/2} = \sqrt{d^2}$$

which is just:

$$\sqrt{d^2} = d$$

Once again, we get the same answer using either method.

Consider:

$$w^{3/4}$$

Using indices laws this is the same as:

$$w^{3/4} = (w^3)^{¼}$$

or in other words:

$$(w^3)^{¼} = \sqrt[4]{w^3}$$

Some Practice Questions

Convert these expressions to terms using just fractions and root symbols – that is, without using negative or fractional indices.

1. $a^3 / a^6 =$
2. $3^{-1}b^5 \times 3^2b^{-6} =$
3. $9^{1/2}m^{1/3} =$
4. $(k^{4/3})^{1/2} =$

Convert these expressions to terms using just negative and fractional indices – that is, without using fractions or root symbols.

5. $^3\sqrt{k} =$
6. $^4\sqrt{y} / y^4 =$
7. $40 / (2h)^2 =$
8. $1 / \sqrt{(5p)^4} =$

Answers

1. $1 / a^3$
2. $3 / b$
3. $3\ ^3\sqrt{m}$
4. $^3\sqrt{k^2}$
5. $k^{1/3}$
6. $y^{-3\ 3/4}$

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7. $10h^{-2}$

8. $0.04p^{-2}$

Take a look at the working out for each answer **here**.

Now What?

This video, along with Indices Laws, covers important concepts in algebra that you will continue to run into throughout mathematics, such as in calculus and more advanced areas. So it's very important that you're familiar with these rules and comfortable using them.

Once you are confident with these rules, why not check your skills in some of the areas covered by the other algebra videos? Look at, for instance, **Factorisation of Algebraic Expressions** or **Algebraic Fractions**.

But When Am I Going To Use This?

Indices are used in many different situations in real life. A common example is in writing very large or very small numbers. These are often written in scientific notation, and can be stored in computers as a type of variable known as a floating point variable. Scientific notation makes heavy use of indices to keep numbers easier to work with. Floating point variables are very important in all areas of computing, including gaming physics.

Indices are also used in the calculation of areas and volumes. For example, the area of a square is the length squared, and the volume of a cube is the length cubed. This is especially important when changing units of measurement, such as from cubic metres to cubic centimetres.

Plus, indices are used in certain kinds of other measurements, including acidity (pH), the loudness of sound (decibels), or the intensity of earthquakes (the Richter scale). All of these measurements use what is known as a logarithmic scale, which relies on indices.

Other Links

Maths is Fun has useful applets to help you understand the basic idea of indices, plus an easy to follow summary of the rules. The first link below is the same as for the topic Indices Laws and also covers negative indices and an index of zero. The next two links cover negative and fractional indices respectively. Sample questions are provided.

- http://www.mathsisfun.com/algebra/exponent-laws.html
- http://www.mathsisfun.com/algebra/negative-exponents.html
- http://www.mathsisfun.com/algebra/exponent-fractional.html

Laerd Mathematics gives a succinct summary of the rules, and follows this up with a wide selection of questions with worked answers available.

- http://mathematics.laerd.com/maths/indices-intro.php

Patrick JMT (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Here are two relevant videos: the first runs through the use of negative indices; the second explains fractional exponents. On both of these pages are links below the video to more videos covering example questions.

- http://patrickjmt.com/negative-exponents/
- http://patrickjmt.com/evaluating-numbers-raised-to-fractional-exponents/