

Before You Watch

This video explains the concept of a mathematical proof and how it is different to a “proof” in other fields. As you will see, the concept of a proof within mathematics is far more rigorous and precise than in other areas or what you probably imagine ‘proof’ to be in your mind. This video doesn’t build on any of the previous topics, so feel free to watch this one straight away.

The Video Content

In mathematics, we often need to prove precisely formulated mathematical statements.

What is a proof?

Mathematical proofs are logical arguments that show the validity of statements.

Proofs must be:

- rigorous
- unambiguous.

In fact, mathematics is distinguished by such proofs, as they are typically not available in other fields.

For instance, in the court of law, one is only expected to prove things “beyond reasonable doubt”.

In the sciences, theories can only be supported by evidence and cannot, in general, be proved absolutely. For example, in medicine, clinical trials can only show that new drugs are likely to be effective with high probability. However, there is always a small

chance that the outcomes of the trial were misleading and that the drug does not work as it should.

Mathematical proofs, in contrast, are absolute and are not subject to statistical uncertainty.

Since mathematical proofs are rigorous and unambiguous, it is also important that the statement to be proved is precise and that all terms used are formally defined.

What is not a proof?

Showing that the statement holds for one particular example is not a proof, as proof needs to hold for all instances of a problem.

For example, the statement:

“The product of any two consecutive integers is even.”

is not proved by:

“Consider integers 5 and 6; $5 \times 6 = 30$, and 30 is even.”

So how could we go about proving this statement?

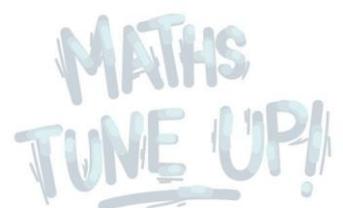
We could try to use a “proof by exhaustion” where we would need to check every pair of consecutive integers. Unfortunately, the number of such pairs is infinite, and we would never finish checking!

Therefore we have to think of another strategy.

In general, constructing a proof requires a lot of:

- perseverance
- creativity.

A dead end is not a signal to give up, but rather a sign that we need to attack the problem differently.



Fortunately, there are some common strategies for proving mathematical statements that we can try on our problem.

Common strategies include:

- direct proof
- proof by contradiction
- proof by construction
- proof by mathematical induction

However, given that this is only a brief introduction to proofs, further discussion of these strategies will be the focus of future topics.

Now What?

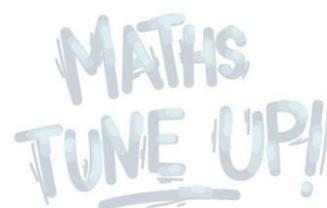
Now that you're familiar with the basic concept of a mathematical proof, the next step is to continue to develop your knowledge through the Other Links below.

Alternatively, take a look at some of the other videos available, such as [Introduction to Algebra](#) or [Introduction to Calculus](#).

But When Am I Going To Use This?

Mathematical proofs are very important in a number of areas, particularly in computer science and simulations. Proofs are also used in research where they can be key to explaining, for example, the difference between an old way and a new way of doing something.

Mathematical proofs, when written down, are a way to keep a record of your understanding and to convey that understanding to others.



Other Links

Maths is Fun introduces a method of proving things called mathematical induction. A simplified explanation is provided along with some clear, basic examples.

- <https://www.mathsisfun.com/algebra/mathematical-induction.html>

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to a list of resources which discuss the application of proofs across a number of different areas. Randomly choose a few that sound interesting to you and start exploring the world of proofs.

- https://www.khanacademy.org/search?search_again=1&page_search_query=mathematical+proofs

Patrick JMT (Just Maths Tutorials) has many video tutorials covering lots of mathematical concepts. This content investigates proof by induction. Start by choosing Example 1 at the bottom of the screen that the link takes you to.

- <http://patrickjmt.com/?s=induction>

