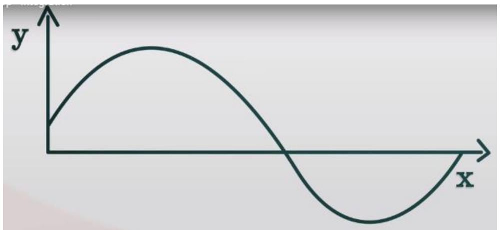
Integration

Before You Watch

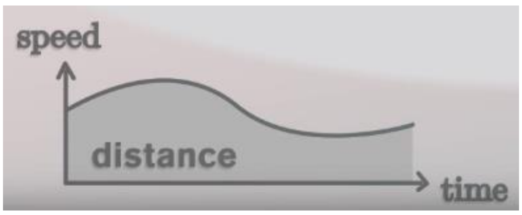
Before watching this video, make sure you've seen **Introduction to Calculus**. In fact, even if you've seen it already, it's a great idea to watch it again! A lot of the notation introduced in that video is used in this one. This topic also builds on the key concept of calculus that was explained in the introduction. So we suggest you watch **Introduction to Calculus**, then come back.

The Video Content

As we saw in Introduction to Calculus, calculus is about the concept of infinitesimal change in one quantity compared to another quantity, for example, temperature with time. Let's look at how we can use this concept to help us find the area under a curve. Consider this curve. The vertical axis is y and the horizontal axis is x.



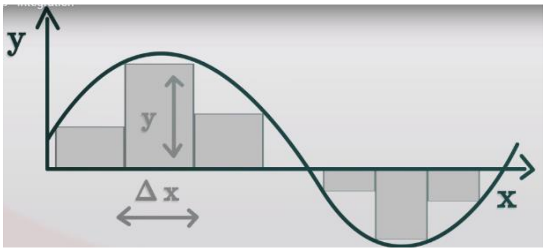
An example of where this is used could be if the y axis is speed and the x axis is time. Then the area under the curve is distance travelled. This helps us determine how far a car has travelled, only knowing its speed over time.



To find the area, we could start by making a series of rectangles under the curve, then add the area of all the rectangles. That would give an approximation of the total area.

From the last video we saw that we often use the capital Greek letter Δ to represent change, so we can express the width of these rectangles as the change in x, Δx.
The height of each rectangle is the value of y at that point.
So:

area of each rectangle = y x Δx

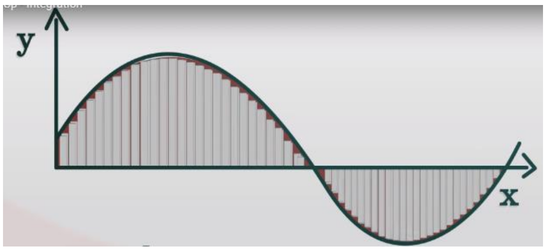


The area of all rectangles together is, therefore, the sum of these rectangles.
The Greek letter Σ is usually used to mean 'sum'.
So:

total area = Σ y x Δx

But we can see that we are missing out on a lot of area. How can we cover more of the area?

If we were to make Δx smaller, the rectangles get narrower. That way, the area gets more accurate and we cover more of the area under the curve.



To get the exact area, we make the rectangles smaller and smaller and,
in doing so, you can see that the area gets closer and closer to the
real area under the curve:

Σ y x Δx

In fact, if we make the rectangles infinitely narrow, the area of the rectangles *is* the area under the curve. Just like in Introduction to Calculus, the Δx is getting infinitesimally small.

To represent an infinitely small step in x, we use dx.

However, as the rectangles become infinitely thin, there also becomes infinitely many of them. So we replace the Σ with a new symbol, the integral symbol $\int$, to represent that it is an infinite sum.

This is called the integral of y times dx:

$$\int y * dx$$

It means the infinite sum of rectangles of height y and width dx.

This is an example of the core concept of integral calculus.

Looking back, we can see that we broke down our complex shape – the curve – into simple shapes we knew the area of. Then we imagined making those simple shapes infinitely small (or narrow, in this case) so that the area of all the simple shapes is the same as the complex shape.

As you will find out, this process has many potential applications in engineering and science.

Now What?

This video builds on **Introduction to Calculus** and discusses the core ideas of integral calculus, which is one of the two main branches of calculus. If you haven't already done so, work through the videos covering the other branch of calculus, differential calculus. Start with **Rates of Change and Differentiation**.

Alternatively, if you've already seen those videos, the next step is to build up your ability to differentiate. You can do this with the Khan Academy at https://www.khanacademy.org/math/differential-calculus/taking-derivatives.

It is easiest to learn how to integrate by first learning to differentiate. This is because integration is the inverse operation to differentiation, in the similar way that division is the inverse operation to multiplication.

But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. It has enormous applications in all areas of engineering and science, and is necessary knowledge to study for a degree in engineering or science.

Specifically looking at integral calculus, integration allows us to move from a rate of change and convert that into an absolute quantity. For example, calculus allows the measurement of a car's speed over time (using it's speedometer) to then be converted into distance (as measured by the odometer). Another example is the measurement of the speed of water flowing through a pipe, which can then be used to calculate the total amount of water that flowed through the pipe.

Other Links

Maths is Fun has a great page that covers the basic concepts of integration. It then extends your learning by demonstrating how to perform the integrations, which is an important skill.

- https://www.mathsisfun.com/calculus/integration-introduction.html

IntMath gives a good explanation of the process of integration, as well as covering the methods of integration. It provides some excellent examples of applications of calculus that are in common use today, and includes applets to help your understanding of both differential and integral calculus.

- http://www.intmath.com/integration/integration-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical and other concepts, as well as questions to test your knowledge. This content provides a whole chapter on taking the derivatives, including of harder equations not covered in this video.

- https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals

Patrick JMT (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This content covers the basic concept of integration. The site also offers a wide selection of other videos covering several integration topics and techniques.

- http://patrickjmt.com/the-definite-integral-understanding-the-definition/