Factorisation of Algebraic Expressions

Before You Watch

This topic covers how to factorise certain algebraic expressions and how to use them. It builds upon a basic understanding of:

- algebra
- how to use and expand brackets
- indices in algebra.

These concepts are explained in **Introduction to Algebra** and **Indices Laws**.
If you're not confident in the above areas, watch these two videos first, to make sure you have the background knowledge needed, then come back.

The Video Content

Our first example is to factorise $2x + 3x^2$.

So what does 'factorise' mean?

Step 1 Understand the question

Factorising an algebraic expression is the opposite process of expanding the brackets. It works in exactly the same way as finding factors of numbers. When you expand the brackets, you multiply the terms outside the brackets with each of the terms inside the brackets to get one expression (which can have + and / or - symbols in it).

For instance, let's expand:

$$4y(2y + x)$$

We do this by multiplying both of the terms in the bracket by $4y$:

$$8y^2 + 4xy$$

So to go from:

$$4y(2y + x)$$

to:

$$8y^2 + 4xy$$

is to expand. To go the other way is to factorise. We can now say that $4y$ and $2y + x$ are factors of the expression $8y^2 + 4xy$.

This process is called factorisation because the equation is being expressed as the multiplication of a series of factors. For example, we can factorise the number 15 by expressing it as a multiple of its prime factors: 15 = 3 x 5.

Back to the original question. How can we factorise $2x + 3x^2$?

Step 2 Develop a plan

The simplest method of factorisation is to find a common factor (amongst each of the terms.) In this case we notice that x goes into both terms. So that's what the plan is: to take x out as a factor.

Step 3 Carry out the plan

To take x as a factor, we need to first multiply and divide by x:

$$x \,/\, x(2x + 3x^2)$$

which doesn't change the value of the equation. Then we manipulate the expression a little to take x out:

$$= x(1 \,/\, x)(2x + 3x^2)$$
$$= x((2x + 3x^2) \,/\, x)$$

Now we can separate this fraction into two fractions, $2x \,/\, x$ and $3x^2 \,/\, x$:

$$= x(2x \,/\, x + 3x^2 \,/\, x)$$

Looking at each of these fractions, we can cross an x off the top and bottom so that our final answer is:

$$x(2 + 3x)$$

That can't be factorised anymore, so we are done. Is our answer correct?

Step 4 Reality check

We can easily check a factorisation by expanding out the brackets. If we get back to the original answer, great! This is because factorisation is the opposite of expansion:

$x(2 + 3x)$
$= x \times 2 + x \times 3x$
$= 2x + 3x^2$

Another example

Say we are asked to factorise $4a^3b^2 + 8ab^4 + 2ab$.

In this case, first we notice that the biggest or highest common factor is $2ab$.
So, as before, we take $2ab$ out the front, and divide everything inside by $2ab$.

Did you know?

In the above example, how do we know that $2ab$ is the highest common factor? Here are the steps.

Step 1

Look at the numbers first. Find the highest common factor (HCF) of the numbers at the front. In this example the numbers are 4, 8 and 2. The highest common factor of these is 2.

Step 2

Look at the letters. A letter is only a common factor if it appears in *all* the terms. For each letter that is in all the terms, we put it to the *lowest* power that appears. In this example, both the letters "a" and "b" appear in all three terms. The letter "a" has powers 3, 1 and 1, so we need to put "a" to the power of 1. The letter "b" has powers of 2, 4 and 1, so we need to put "b" to the power of 1.

Step 3

Put it all together. We can see that our highest common factor is the number 2, and the letters "a" to the power of 1 and "b" to the power of 1. So the highest common factor is $2ab$.

So, working with the highest common factor, take $2ab$ out the front, and divide everything inside by $2ab$:

$$2ab((4a^3b^2 + 8ab^4 + 2ab) / 2ab)$$

Now break up that fraction into three fractions:

$$2ab((4a^3b^2 / 2ab + 8ab^4 / 2ab + 2ab / 2ab)$$

Then cross off all the common terms top and bottom for the three different fractions. The final result is:

$$2ab(2a^2b + 4b^3 + 1)$$

We've reached the point where the term inside the bracket can't be factorised any more. The easiest way we can see this is because one of the terms in the bracket is 1. The number 1 has no factors other than itself, so we can't factorise it further.

Can you do the reality check? Do you get to the original expression if you expand this bracket?

Multiply each term inside the brackets by $2ab$:

$$2ab(2a^2b) + 2ab(4b^3) + 2ab(1)$$

Then expand:

$$4a^3b^2 + 8ab^4 + 2ab$$

This matches the original expression so, yes, the answer is correct.

Some Practice Questions

Factorise the following expressions:

1. $3k^8 + 2k^4$
2. $4a + 4b$
3. $b^2j^7 + j^5m^3$

4. $6k^2 + 12k^3m$

5. $15p^4t^3y^2 + 20k^3p^4z^3$

6. $2a^2b - 2^8ab$

7. $4x^2 + 8xy$

8. $6m - 3mn + 12km$

Answers

1. $k^4(3k^4 + 2)$

2. $4(a + b)$

3. $j^5(b^2j^2 + m^3)$

4. $6k^2(1 + 2km)$

5. $5p^4(3t^3y^2 + 4k^3z^3)$

6. $2ab(a - 14)$

7. $4x(x + 2y)$

8. $3m(2 - n + 4k)$

Take a look at the working out for each answer **here**.

Now What?

The method presented here is the simplest method of factorisation – identifying a common factor. Two other methods often used are factorisation by grouping, and the difference of two squares. Factorisation is a valuable skill and

it's useful to also become familiar with these other methods. Further resources are provided in the Other Links section.

Once you are confident with factorisation using the method covered here, you can examine other algebraic skills, such as **Algebraic Fractions** or **Linear Equations**.

Other Links

If you have difficulty finding the highest common factor in algebraic expressions, this video tutorial at **Virtual Nerd** gives a very thorough method of determining the highest common factor. The site also has other useful tutorial videos.

- http://www.virtualnerd.com/algebra-2/quadratics/solve-equations-by-factoring/factoring-strategies/greatest-common-factor-example

Everything Maths has a good summary of different factorisation methods as well as a wide selection of worked examples and questions with answers.

- http://everythingmaths.co.za/maths/grade-10/01-algebraic-expressions/01-algebraic-expressions-06.cnxmlplus

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the relevant section. At the top of the page is the topic “Introduction to Factorization” followed by further topics on factorisation.

- https://www.khanacademy.org/math/algebra/polynomial-factorization

Patrick JMT (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Patrick JMT uses the term “factoring” and there is a very wide selection of factoring videos available covering numerous methods.

- http://patrickjmt.com/