Exponentials and Logarithms

Before You Watch

This topic introduces solving equations where the letter you are asked to find the value of is in the index (also called the exponent). This requires the use of logarithms. The definition of a logarithm is:

if:

$$a^{b} = c$$

then:

$$\log_{a} c = b$$

When we read the line above, we say “log to the base ‘a’ of ‘c’, is equal to 'b'”.

Another way we can think about logarithms is as the inverse of the exponential. For example, if we have a number x, and we put a number, say, 5 to the power of x:

$$5^{x}$$

then we put that to the log of base 5:

$$\log_{5}(5^{x})$$

that is equal to x:

$$\log_{5}(5^{x}) = x$$

In other words, if we start with x, and then put 5 to the power of x, we can think of the log process as “undoing” the process of putting 5 to the power of x. So these two processes are the inverse of each other, in a similar way to multiplication and division being the inverse of each other (that is, if you start with x, then multiply by 5, then divide by 5, you get back to x).

Need more of an introduction to the nature of logs? Look at one of the links below before watching this video.

- https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms
- https://www.mathsisfun.com/algebra/logarithms.html

As well as having an understanding of the nature of logs, it is useful to revise your knowledge of the number "e" before watching this video. Remember, "e" is an irrational number that has many real world applications, and that's why it has been given a special name. In fact, "e" is similar to the number π, which is also an irrational number that has a special name because of its applications. The number "e" is approximately equal to 2.718, however, most calculators have a dedicated button for it.

Comfortable with using logarithms and the number "e"? Then you're ready for this topic!

The Video Content

This content explores how to solve equations using logarithms. You might recall from high school that just as division is the inverse of multiplication, logs are the inverse of exponentials: for example, 2^5 is an exponential. Logarithms are very useful in mathematics. A very important property of logarithms is that the log of some number – let's call it r – to the power of some other number – say p – is equal to p times the log of r. For example:

$$\log(7^5) = 5 \log(7)$$

Say we are asked to solve:

$$5^p = 16$$

Step 1 Understand the question

What the question is really asking us to do is rearrange this equation and have p on its own on one side, with what it equals on the other.

5 to the power of p equals 16, so we need to find the value of p such that 5 to the power of that value gives 16. We already know 5 to the power 2 = 25 and 5 to the power 1 = 5, so p must be somewhere between 1 and 2.

Here's a hot tip: doing this kind of quick estimate in your head can be a very effective way of catching mistakes!

Step 2 Develop a plan

Remember that we always do the same to both sides of an equals sign. Here we need to take the log of both sides, solving the equation by using the properties of logs. That's the plan.

Step 3 Carry out the plan

Taking the log of both sides gives us:

$$\log(5^p) = \log(16)$$

Using the property from the start of the topic, we know:

$$\log(5^p) = p \times \log(5)$$

So we have:

$$p \times \log(5) = \log(16)$$

Divide both sides by log 5 and we get:

$$p = \log(16) / \log(5)$$

Now we have something we can enter into a calculator. It doesn't matter which base we use, either base 10 or base e, it'll give the same answer regardless. In this case p is approximately 1.7 or, with more decimal places, 1.7227.

Step 4 Reality check

We know that p should lie between 1 and 2, so the answer already seems reasonable. To make sure it is correct, we can take the answer, p = 1.7227, and put it back into the equation:

$$5^p = 16$$

and see if the left and right sides of the equation match.

It won't match exactly since we used an approximation for p, but it should be close. Give it a try. See if:

$$5^{1.7227} = 16$$

Putting $5^{1.7227}$ into the calculator gives a result of:

$$5^{1.7227} = 15.9998$$

rounded off to 4 decimal places.

Obviously this isn't exactly the answer 16 from our question, but remember that 1.7227 was already rounded off from what was on the calculator, therefore we don't expect the answer to be exactly 16.

The number we got, however, is so close to 16 (in fact, it is 16, rounded off to three decimal places) that we can be confident this is the correct answer.

Some Practice Questions

1. Solve for m $2^m = 8$
2. Solve for u $5^u = 78{,}125$
3. Solve for t $e^t = 15$
4. Solve for n $4^n = \pi$

Answers

1. $m = 3$
2. $u = 7$
3. $t = 2.708$ (to 3 decimal places)
4. $n = 0.826$ (to 3 decimal places)

Take a look at the working out for each answer **here**.

Now What?

This topic explains how to solve an important category of equations, where the letter is in an index. It will also have refreshed your memory of logarithms and the number "e".

Once you are confident in your ability to solve equations like this, check your skill at solving other types of equations, such as **Quadratic Equations** or **Simultaneous Equations**.

But When Am I Going To Use This?

Exponentials are often found in situations where the rate that something grows is related to the size it already is. A very simple example of this is a home loan: the bigger the loan or interest rate, the faster it grows. This type of growth is found naturally as well, for instance, in bacterial growth.

Logarithms are also commonly used to measure things over a very large scale, where the values getting measured can be very large and very small. For example, the decibel, the standard unit for measuring the volume of sounds, is a logarithmic scale. Another example of a logarithmic scale is the Richter scale, the standard scale for the measurement of the strength of earthquakes.

Other Links

Fort Bend Tutoring features a series of YouTube videos covering many different mathematical concepts. The video below covers solving exponential equations, and also demonstrates how to solve exponential equations in situations where you don't need to use the logarithms.

- https://www.youtube.com/watch?v=Y-PaFgFDLZk

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. Look at the top of this resource for the link to the chapter that covers logarithms. Half way through the chapter is a video that shows you how to solve the exponential equation.

- https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/exponential-equation

Patrick JMT (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. The video linked below demonstrates how to solve exponential equations.

- http://patrickjmt.com/solving-exponential-equations/