Differentiation of Polynomials

Before You Watch

This video continues directly on from **Rates of Change and Differentiation**, in which the concept of a rate of change was introduced and we investigated the need to know the value of instantaneous rates of change. The term 'to differentiate' was also discussed. Differentiation is the process of calculating a rate of change. This video describes how to differentiate a specific class of equations known as polynomials. So make sure you've seen **Rates of Change and Differentiation** recently before watching this video.

This topic also builds upon earlier algebraic concepts such as **indices**, including **negative indices**, and **linear equations**. It is important to be comfortable with algebra and manipulating algebraic equations before continuing with calculus, so watch those videos again if you need to, then come back.

The Video Content

This topic builds on the previous calculus videos, and looks at how to differentiate a polynomial.

Let's say a car's position p is given by $3t^2$. So:

$$p = 3t^2$$

How do we calculate the speed?

Step 1 Understand the question

As we saw from the last video, to calculate the speed of a car, we need to calculate the rate of change between position and time.

Step 2 Develop a plan

To calculate the rate of change, we need to differentiate the equation. That's the plan.

Step 3 Carry out the plan

How to find the derivative of an equation depends on what kind of equation is it.
The equation $p = 3t^2$ is a polynomial.

Here are some examples of polynomials:

- $p = 3t^2$
- $j = 6r + 3r^2$
- $m = n^3$
- $y = 2x^2 + 7x + 3$

They have two different letters. One letter is on its own on one side of the equation and all the terms on the other side have the other letter, or just a number. The terms on the other side can have a number in front of the letter, and they can also have any integer power.

Did you know?

For an equation to be a polynomial, the power must be a positive integer. If there are any negative powers then it is not a polynomial.

To differentiate a polynomial, first we'll look at what happens to the left hand side.

As we saw in the last topic, differentiating calculates the rate of change. So instead of just position, we have change in position over change in time. In other words:

$$dp / dt$$

Now we can differentiate the right hand side of the equation.

To do this, take the power and bring it out the front to multiply our expression with. The 3 and the t are left alone. Subtract 1 from the power, so the power changes from 2 down to 1:

$$p = 3t^2$$
$$dp / dt = 2 \times 3t^1$$

We can then simplify that, so we get:

$$dp / dt = 6t$$

That's our answer. The speed of the car is given by $dp / dt = 6t$

Let's do another example.

This time we will differentiate:

$$y = 4x^7 + 2x^3 + x$$

The process is the same as before. The letter on its own is now y, and the letter on the other side is x.

So when we differentiate the y becomes dy / dx.

On the right hand side we deal with one term at a time.

Take the 7 down, leave the $4x$ as is, and reduce the power by 1, down to 6.

For the next one we pull the 3 down, leave the 2x, and reduce the power by 1.

For the last one we remember that x means x to the power of 1, so we pull the 1 out the front, and reduce the power to 0.

From the indices video we saw that anything to the power of zero is 1, so our last term just becomes 1:

$$y = 4x^7 + 3x^3 + x$$

$$dy / dx = 7 \text{ x } 4x^7 \quad + \quad 3 \text{ x } 2x^2 \quad + \quad 1 \text{ x } x^0$$

Simplifying, we see our answer is:

$$dy / dx = 28x^6 + 6x^2 + 1$$

Done! You have found the derivative of a polynomial.

Some Practice Questions

Differentiate:

1. $y = x^2$
2. $p = 4t^2 - 3t$
3. $j = 5r^6 - 7r^4 + 5r^2$
4. $q = (7x + 3x^2)^2$ (Hint: expand out the brackets first)

Answers

1. $dy / dx = 2x$
2. $dp / dt = 8t - 3t$
3. $dj / dr = 30r^5 - 28r^3 + 10r$
4. $dq / dx = 36x^3 + 126x^2 + 98x$

Take a look at the working out for each answer **here**.

Now What?

By now you will be familiar with the three videos that introduce the differential branch of calculus: **Introduction to Calculus**, **Rates of Change and Differentiation** and this topic, **Differentiation of Polynomials**. You should understand the core concepts of calculus and know what a rate of change is. You will also know that differentiation is all about calculating the rate of change, and know how to differentiate one category of equations, the polynomials. From here there are two main directions you can go.

One option is to explore how to differentiate other types of equations, such as those involving trigonometry, or exponentials. To do this you should consider looking at sites such as the Khan Academy at https://www.khanacademy.org/math/differential-calculus/taking-derivatives

Alternatively, you could investigate the other branch of calculus, integral calculus. This is introduced in the topic **Integration**.

But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. For instance, velocity (or speed) is a change of position over a change in time, and acceleration is a change in velocity over a change in time – so any motion is studied using calculus. Other examples include the flow of water through pipes over time, or changing commodity prices against demand. Because change is everywhere, the potential applications for calculus are endless, particularly in engineering and science. Calculus is necessary knowledge for any degree related to engineering or science.

Other Links

Maths is Fun has a great page that takes you through a simple problem which highlights the need for calculus to discuss changes happening around us. It then continues to explore the main two areas of calculus, differentiation and integration, and provides regular questions to test your understanding.

- https://www.mathsisfun.com/calculus/introduction.html

IntMath gives a bit of historical perspective to explain the sometimes confusing notation that is used in calculus, discussing how it is the mixed product of two mathematicians working independently. It also provides some excellent examples of applications of calculus that are in common use today, as well as helpful applets to understand both differential and integral calculus.

- http://www.intmath.com/calculus/calculus-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical and other concepts, as well as questions to test your knowledge. This content provides a whole chapter on taking the derivatives, including of harder equations not covered in this video.

- https://www.khanacademy.org/math/differential-calculus/taking-derivatives

Patrick JMT (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This content runs through differentiation of simple polynomials, but the site also provides videos demonstrating more complex differentiation.

- http://patrickjmt.com/basic-derivative-examples/