

Before You Watch

This video builds directly on the concepts discussed in Introduction to Trigonometry, which presented the three basic trigonometric ratios: sin, cos and tan. If you haven't watched the [Introduction to Trigonometry](#) video yet, do that first, then come back.

The Video Content

Think about a ball being thrown. The ball moves up as you throw it, and it also moves across the field. To predict the path of the ball, we need to break up the movement into two parts: the vertical component, up and down, and the horizontal component, across the field.

This is called breaking down the movement into perpendicular components. It is used all the time in engineering and physics and is an excellent example of a common use of trigonometry.

Step 1 Understand the question

Let's say that you throw a ball at 120 km/hr, at an angle of 22° above the horizontal. The question is: what are the vertical and horizontal components of this initial motion?

Step 2 Develop a plan

Our starting throw has a length of 120 and is 22° above the horizontal.



These two lines, one horizontal and one vertical, are called the vertical and horizontal components of our starting throw. We need to find these two lengths, x (horizontal) and y (vertical).

That's the plan.

Step 3 Carry out the plan

Remember in trigonometry, the sine of an angle is equal to the length of the opposite side over the length of the hypotenuse.

So:

$$\begin{aligned}\sin 22^\circ &= \text{opposite} / \text{hypotenuse} \\ &= y / 120\end{aligned}$$

To get the y on its own, multiply both sides by 120 and cross off the 120 top and bottom on the right.

This gives us:

$$120 \sin 22^\circ = y$$

Similarly for the horizontal, cosine is the adjacent side over the hypotenuse.

Therefore:

$$\cos(22^\circ) = x / 120$$

As before, multiply both sides by 120, cross off 120 top and bottom and we have:

$$120 \cos(22^\circ) = x$$

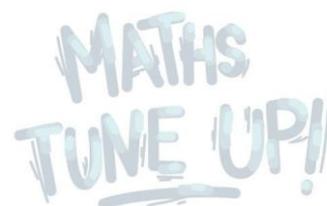
Now we put this into the calculator and get:

$$y = 44.95 \text{ (to 2 decimal places)}$$

and

$$x = 111.26 \text{ (to 2 decimal places)}$$

So, our initial throw of 120 km/hr consists of moving across the field at 111.26 km/hr and vertically up at 44.95 km/hr. Of course, gravity will pull the ball down, so this will change over time.



Did you know?

Whenever doing trigonometry, it is very important to check that your calculator is in the correct mode, depending on what the angles are being measured in. In this example, we are measuring the angles in degrees, so we must make sure that our calculator is in degrees mode. The next most common way of measuring angles is in Radians, but for this example it's important that our calculator is not in Radians mode. If it is, our answer will be wrong.

Step 4 Reality Check

Do these numbers make sense? At a glance, the numbers seem ok; neither of them seems too big, nor too small to be unreasonable. If we wanted to be very sure, we could check using Pythagoras' theorem. Give it a go and see if it adds up!

Put our two answers into Pythagoras' theorem:

$$\sqrt{(44.95^2 + 111.26^2)}$$

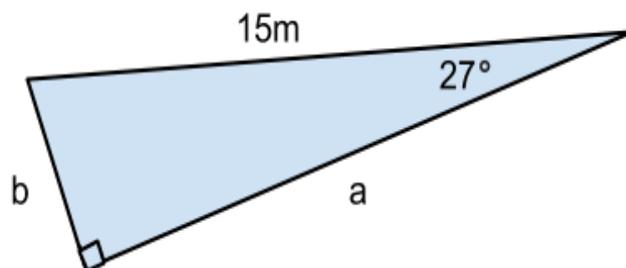
Put this into the calculator and we get:

$$\sqrt{(44.95^2 + 111.26^2)} = 119.9970 \text{ (to 4 decimal places)}$$

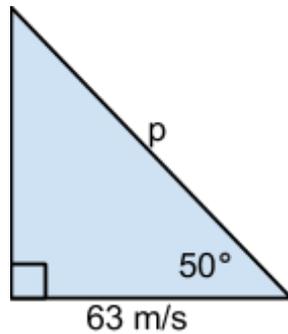
We don't expect to get *exactly* our original speed of 120, because we rounded off our answers to 2 decimal places. However, the answer is very close to 120, so we can be more confident that our answer is reasonable.

Some Practice Questions

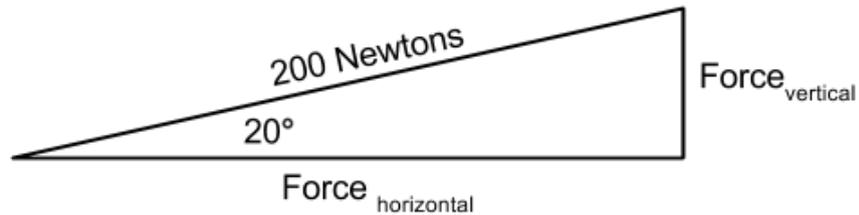
1. Find the lengths of the sides a and b , of the following triangle:



2. Find the length of the side p of the following triangle:



3. Consider a box is being pulled by a rope with a force of 200 Newtons. The rope is angled 25° above the horizontal. In other words, the force acting on the box can be visualised by the triangle below. What is the horizontal component of this force?



4. If a bullet is fired at 400 m/s at 10° above the horizontal, what is the vertical component of its velocity?

Answers

1. $a = 13.37\text{m}$, $b = 6.81\text{m}$ (2 decimal places)
2. 98.01 m/s (2 decimal places)
3. 187.94 N (2 decimal places)
4. 69.46 m/s (2 decimal places)

Take a look at the working out for each answer [here](#).

Now What?

After watching this video, plus **Introduction to Trigonometry**, you should be comfortable with finding the length of a side of a right-angled triangle when given the length of another side and an angle of the triangle.

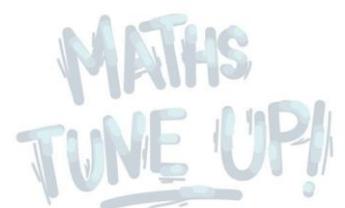
Beyond this, it is recommended that you investigate other applications of trigonometry, such as finding an angle when given the sides of the triangle, or the sine and cosine rules. You can explore these through websites such as <https://www.khanacademy.org/math/trigonometry/basic-trigonometry>.

Alternatively, you can learn about other algebraic concepts such as **Algebraic Fractions** or **Factorisation of Algebraic Expressions**.

But When Am I Going To Use This?

The obvious application of trigonometry is for right-angled triangles. This is common in construction, surveying and the application discussed in this video, where things like forces, or movement, are broken down into perpendicular components. However, trigonometry can also extend to non-right angled triangles through, for example, the sine rule and the cosine rule.

Moving beyond triangles, trigonometry is critical to the study of waves, such as radio waves. This is very important in fields such as wireless communication and quantum physics: for instance, mobile phone technology would be impossible without a method of breaking down a signal into a series of sines and cosines known as Fourier Analysis.



Other Links

Maths is Fun has a great page summarising the trigonometric ratios and includes several different applets to help you visualise and understand the basics. It also has other pages dealing with more advanced trigonometry subjects to develop your understanding.

- <https://www.mathsisfun.com/algebra/trigonometry.html>

The **Mathspace** page on trigonometry is well laid out with a variety of instructive videos, applets and quizzes. It also has apps for iPad, iPhone, Android and Windows Phone. It does require you to create a login, however, using these resources is free.

- <https://mathspace.co/learn/world-of-maths/trigonometry/trigonometric-ratios-6831/special-ratios-258/>

GeoGebra is a mathematics app that works on a wide variety of platforms, including tablets, and in a web browser. It is used to create great tools for teaching and learning. The link here is an example of a GeoGebra 'program' that helps demonstrate how trigonometric ratios stay constant, no matter how the triangle shrinks and expands.

- <http://www.geogebra.org/student/b77950#material/11887>

The following link is to a simple animation which demonstrates the creation of the graph of the sine function using the unit circle. The source code for the creation of the animation is also attached. For people with a natural orientation towards computer programming (which is a huge application of mathematics) this may help you understand the nature of trigonometry.

- <http://www.humblesoftware.com/demos/trig>

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the chapter covering basic trigonometry, and continues onto more advanced trigonometric concepts.

- <https://www.khanacademy.org/math/trigonometry/basic-trigonometry>

Patrick JMT (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. This video covers the trigonometric ratios and solves a variety of problems using these ratios.

- <http://patrickjmt.com/right-triangles-and-trigonometry/>

